S. S. Narayanaswami

Markov Stochastic Modelling for City Size Mobility
Mobility

1. Introduction

THE spatial distribution of people in a country reflects its economic activity, material possession and prosperity. Over a five or ten year period the demographic situation of a country is usually affected to a greater degree by changes in proportional distribution of its people than its growth. The rapidity and magnitude of differential impact of modem development are so complex that the real mechanics of adjustment of people to economic opportunities comes more from the distribution by movement than by natural growth. This problem has become a serious concern of the planners because an unplanned concentration of people in cities may take away much of the benefits accrued by planned development.

In the analysis of urbanization, the total population is usually dichotomised **into r**ural-urban classification. In the recent past, urbanization in developing countries is weighted in favour of big cities and there is no known or established pattern of distribution of people. The urban population can be polytomised by stratifying the cities according to number of inhabitants. The hidden differences in the distribution can be identified by classifying the population in places by different sizes-. An abundant empirical measures are being employed to know the hierarchical classification of cities by size. In this analysis the Marrov Model is employed to know the mobility of the cities by size.

2. The Markov Model

Urbanization is a processor concentration of people in special areas regarded as urban. As demographic transition progresses not only the concentration points increases but also the number of inhabitants in different size classes enlarges. Most of the city size distribution (CSD) analyses are directed to identify either the increases in the number of urban places or differences in the size classes at a particular point of time or over a period of time, Markov Model introduced by the Russian Mathematician A. A. Markov in 1907 provides an opportunity for simultaneous analysis of urban growth, vertical and horizontal.

Let the urban population of a country be divided into *m* size classes where *m* stands for the states of a Markov Chain (MC), that is, different size classes **are** termed as the states of a MC. As population growth takes place cities move from one state to another at different points of observation. These points can be taken as the natural points of census dates which are normally conducted in most of the countries within a period of ten years. At different points of time the changes in city size distribution can be represented by the transition probability matrix

$$P = \begin{bmatrix} P_{11} & P_{12} & \ldots & P_{1j} & \ldots & P_{1m} \\ P_{21} & P_{22} & \ldots & P_{2j} & \ldots & P_{2m} \\ \ldots & \ldots & & \ldots & \ldots & \ldots \\ P_{i1} & P_{i2} & \ldots & P_{ij} & \ldots & P_{im} \\ \ldots & \ldots & & \ldots & \ldots & \ldots \\ P_{m1} & P_{m2} & \ldots & P_{mj} & \ldots & P_{mn} \end{bmatrix}$$

$$P_{ij} > 0 \ \forall \ i, j, \quad \text{and} \quad \sum_j P_{ij} = 1, \quad i, j = 1, 2, \ldots m. \tag{2.1}$$

The nicer properties of this matrix provide an opportunity to estimate the future distribution of cities under different size classes.

$$P(X_n = j / X_1 = 1, \quad X_2 = 2, \ldots X_{n-1} = i) \\ = P(X_n = j / X_{n-1} = i). \tag{2.2}$$

An intuitive interpretation of a MC is simple that the probability of going from the *i*-th class to *j*-th class does not depend on how we got to the *j*-th class. Only the present state of the process determines the future.

$$P(X_n = j) = \sum_i P(X_{n-1} = i) \, P(X_n = j / X_{n-1} = i) \\ = \sum_i P_j^{(n-1)} P_{ij} = P_j^{(n)}. \tag{2.3}$$

The probability that the process is in size class j at the n-th point of observation is equal to sum over all probabilities that the process is in size class i at the $(n-1)$ st step, on the next step the process moves from size class i to size class j. For $n = 1$

$$P_j^{(1)} = P_j^{(0)} P_{ij},$$

where $P_j^{(0)}$ is the initial probability, constituting the probability vector

$$C_0 = (P_1^{(0)} P_2^{(0)} \ldots P_m^{(0)})$$

for $n = 2$

$$P_j^{(2)} = \sum_i P_j^{(1)} P_{ij}$$

where $P_j^{(1)}$ is the j-th element of the first step transition probability which constitutes the probability vector

$$C_1 = (P_1^{(1)} P_2^{(1)} \ldots P_m^{(1)})$$

implies by induction

$$C_n = C_{n-1} P \quad \text{for} \quad n \geqslant i \tag{2.4}$$

$$C_n = C_{k-1} P \ldots = C_0 P^n. \tag{2.5}$$

Under the assumption that the transition probability remains the same for a considerable length of time which are known as homogeneous MC of which we are presently confined to, an obvious question arises what distribution of cities among different size classes can be expected at any subsequent period say after fifty or sixty years !

Relationship (2.5) provides a solution that the distribution at time n as dependent on the initial distribution (C_0) and on the transition matrix P raised to n-th power. It is thus clear that a random process that results in a sequence of states that are Markovian, with stationary transition probabilities, is completely specified when we know the initial probability density and the transition probabilities. It is well known that for any matrix of kind P, the distribution of the city sizes tends in time to a value $\overline{C}$, which is independent of original distribution. The distribution $\overline{C}$ is here called equilibrium distribution, satisfying

The probability that the process is in size class j at the n-th point of observation is equal to sum over all probabilities that the process is in size class / at the (n — 1) st step, on the next step the process moves from size class i to size class j For n = 1

$$P_j^{(1)} = P_j^{(0)} P_{ij},$$

where $P_j^{(0)}$ is the initial probability, constituting the probability vector

$$C_0 = (P_1^{(0)} P_2^{(0)} \ldots P_m^{(0)})$$

for $n = 2$

$$P_j^{(2)} = \sum_i P_j^{(1)} P_{ij}$$

where $P_j^{(1)}$ is the j-th element of the first step transition probability which constitutes the probability vector

$$C_1 = (P_1^{(1)} P_2^{(1)} \ldots P_m^{(1)})$$

implies by induction

$$C_n = C_{n-1} P \quad \text{for} \quad n \geqslant i \tag{2.4}$$

$$C_n = C_{k-1} P \ldots = C_0 P^n. \tag{2.5}$$

Under the assumption that the transition probability remains the same for a considerable length of time which are known as homogeneous MC of which we arc presently confined to, an obvious question arises what distribution of cities among different size classes can be expected at any subsequent period say after fifty or sixty years !

Relationship (2.5) provides a solution that the distribution at time n as dependent on the initial distribution (C"$_0$) and on the transition matrix P raised to n-th power. It is thus clear that a random process that results in a sequence of states that are Markovian, with stationary transition probabilities, is completely specified when we know the initial probability density and the transition probabilities. It is well known that for any matrix of kind P, the distribution of the city sizes tends in time_to a value $\bar{C}$, which is independent of original distribution. The distribution C is here called equilibrium distribution, satisfying the relationship

$$\bar{C} - P\bar{C} \qquad (2.6)$$

and once this distribution is reached, it will be maintained through time. The distribution so reached can be seen to depend on the structural propensities of CSD and not on the distribution of cities among different size classes found at any instant.

3. Mobility Index

In an urban population, the movements of cities between size classes will take place as individual cities cross the upper limits of the class interval to which it belongs at different points of observation. The forces that operate for the movements of a city from one size class to another are multivarious. Apart from natural growth, it can be due to geographical location, climatic condition, •Economic activity, governmental policy, rural-urban and intercity migration. la complete absence of these forces, the movements of city from one class to another will be highly restricted to natural growth. The eventful happening of these forces explicitly will produce greater growth of population resulting frequent crossing. In the absence of natural growth, migration (rural-urban, inter-city) will produce the transitions from one size class to another. Hence an index to identify the mobile nature of cities will give us strength to study the internal structure of the CSD of two countries at a particular point of time or of a single country at different points of time. Numerous descriptive measures of mobility which depend on transition proportions have been devised for empirical work but the interest is currently restricted to use of transition matrix, to measure the mobility of CSD.

The transition matrix P used to describe the movements of city-size can be used to identify a summary measure for the mobility of CSD. These measures lire analogue to the indexes developed by Prais (1955), Bartholomew (1973) for measuring social mobility and by Adelman (1961) for measuring industrial mobility. The important points brought out here are (1) to make these indexes at home to measure city size mobility, (2) demonstration of the use of fundamental matrix to measure mobility of City Size in the case of an absorbing MC. It happens in the case of urban population, more so in the case of growth transition matrices, that all the cities will eventually move into a single largest class, however long, the projections need be.

The]average number of generation (each generation, being the number **of**

years between two Census points) spent in a particular class serves as a basic measure to study the mobility of CSD. In *a highly* immobile society the average period of stay at different size classes will be more, compared to the average number of generation spent in different classes by a mobile urban population.

The average number of generations spent in a size class may be calculated **as** follows. Let $C_j^{(0)}$ be the number of cities in class *j* at the initial period. The number $C_J^{(0)} P_{ii}$ will be found in the y-th class at the next census, **will** be found in the third point of observation and so on. The total time spent in the *j* th class by *Cj* cities at present is

$$T = C_j^{(0)} + C_j^{(0)} \; P_{jj} + \ldots C_j^{(0)} \; P_{jj}^m + \ldots$$

on dividing by $C_j^{(0)}$ we get the average time,

$$t_j = 1 + P_{jj} + P_{jj}^2 \; + = \frac{1}{1 - P_{jj}} . \qquad (3.1)$$

Unless comparison is made with a standard population the hidden meaning of these indexes can't be given a clear empirical interpretation. There are in principle an infinite number of perfectly mobile CSD which may be used for comparison. The equilibrium distribution which will be reached by our partir-cular group of cities can be a serviceable substitute. This distribution will have the transition matrix as

$$T = \begin{bmatrix} t_1 & t_2 & \ldots & \ldots & t_m \\ t_1 & t_2 & \ldots & \ldots & t_m \\ \cdot\cdot & \cdot\cdot & \cdots & \cdots & \cdot\cdot \\ t_1 & t_2 & \ldots & \ldots & t_m \end{bmatrix} .$$

The index provided by Prais will become

$$m_j^* = \frac{1 - t_j}{1 - P_{jj}} , \qquad = 1, 2, \ldots k,$$

and Chat of Adelman will become,

$$I^n = \frac{\sum_{j=0}^{k} \frac{t_j}{1 - t_j}}{\sum_{j=0}^{k} \frac{c_j^n}{1 - P_{jj}}} .$$

The equilibrium distribution taken up for comparative purposes has many drawbacks of physical interpretation. Before the equilibrium is reached, the internal structure of the system ought to have changed because of the dynamics of human adjustments to changing conditions. Moreover, there is no point in comparing with a distribution which will be realised after 200 or 300 years apart, Prais himself has suggested, "If there were other information available on long term trends in the occupation distribution of society it may be preferable to use some other distribution in the place of equilibrium distribution suggested here." Preston and Bell (1961) used the initial configuration of the distribution as a perfectly mobile distribution apart from giving weights to each size class. The revised index for time *n* as given by them are,

$$M^n = \sum_{j=0}^{k} \frac{1 - P_{ij}}{1 - s_j^n} \; w_j^n$$

where *Pij* is the probability of a firm in each size class remaining in that class, S_j the relative frequency of times in each class at time *n* and *Wi* the share of each class in the total size of the group at time n.

A more direct measure was provided by Bartholomew (1973) by counting the class boundaries crossed in passing from one point of observation to the next. If C_1, c_2, .. . C_k, are the number in the classes of a given census the expected number of class boundaries crossed in moving to the next point is

$$\sum_{i=1}^{k} \sum_{j=1}^{k} C_i P_{ij} \mid i - j \mid .$$

To avoid the difficulties of base line generations C_1 can also be replaced by the equilibrium distribution, P_t.

A preview of the transition matrices presented in Table 2 tells us that the indexes reviewed above breakdown as we are observing an absorbing Markov Chain. It is but natural to expect all the cities to move into one single class however long the waiting time need be. As these transition matrices depict growth matrices, we believe that the empirical evidence is highly reflective of a real system in which after a certain number of points of observation all cities will move on to the largest class. Under these circumstances, the fundamental matrix of an absorbing MC gives us the necessary impetus to measure the mobility of CSD, through the average number of generations spent by different size classes at different states before being absorbed in the largest class,

Given a transition matrix *P*, for a MC, we rearrange it by row and column operations, so that we obtain a partition of *P* into four submatces one of* which contain all the ergodic states and the another containing transient states. The matrix *P*, becomes

$$P_{k,\,k} = \begin{bmatrix} E_{k-r,\,k-r} & O_{k-r,\,r} \\ G_{r,\,k-r} & T_{r,\,r} \end{bmatrix}$$

where *E* is a *(k* — r) *(k — r)* matrix dealing with the process after it has reached an ergodic set, *O* is *(k — r}* (r) is a zero matrix, G is a (r) *(k* — r) matrix of transition probability from transient states to ergodic states and r is a (r) (r) matrix of transition probabilities between transient states. The above partition in the case of a finite MC becomes

$$P_{k,\,k} = \left[\begin{array}{c|c} I & O \\ \hline G & T \end{array}\right]$$

where 7 is a (m — r) (m — *r)* identity matrix. The matrix *F*, given $F = (I - T)^{-1}$ is known as fundamental matrix of an absorbing MC yields the average number of times that a random process is in a given transient state *j*, given that the process started at state /. Additional questions concerning the absorbing MC can also be dealt with the fundamental matrix. Of these, the one which provides variance of the number of times that a process is in transient state may help us to know more about the mobility of CSD. It is identified as

$$V(m_j/i) = F(2D - I) - s.$$

where F *is* the fundamental matrix, D is an (r) (r) diagonal matrix containing the main diagonal elements of the fundamental matrix, I is a (r) (r) identy matrix, 5 is a *(r)* (r) matrix with elements as squares of the elements of the fundamental matrix.

As in the case of ordinary statistical analysis, we can also calculate the coefficient of variation for the mean number of times spent by various states of an absorbing MC.

Let

$$M = \begin{bmatrix} m_{11} & m_{12} & \ldots & m_{1j} & \ldots & m_{1r} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ m_{r1} & m_{r2} & \ldots & m_{rj} & \ldots & m_{rr} \end{bmatrix}$$

and

$$V = \begin{bmatrix} V_{11} & V_{12} & \ldots & V_{1j} & \ldots & V_{1r} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ V_{r1} & V_{r2} & \ldots & V_{rj} & \ldots & V_{rr} \end{bmatrix}$$

be the mean and variance of the number of times that a process is in a transient state t. The coefficient of variation for the number of times spent in different states for an absorbing MC is

$$C\cdot V\cdot (m_{ij}/i = \frac{(V_{ij})^{\frac{1}{2}}}{m_{ij}} \times 100, \forall\, i, j = 1, 2, \ldots r.$$

It is evident from the above that the fundamental matrix Fof an absorbing MC provides tlit accessary impetus to study the mean, variance and coefficient of variation of the number of transitions before absorption.

4. Data Analysis

The data for this analysis are taken from Census of India (1971), Tamilaadu, a state of Indian Union, enjoying a very traditional urban compared to other parts of the country. Indian Census has a history of more than hundred years and the period of observation is confined to 1901 to 1971 omitting the data from the initial period of censuses for obvious reasons of coverage. As we are dealing with 'number of urban places' the quality will be of very high order compared to any other demographic characteristics and at least in this respect, we are in a confident position to exploit the census materials as applicable to our study.

As per latest census classification, the definition adopted for urban areas is as follows:

(A): All places with municipality, cantonment/corporation or notified town area

(B): All places which satisfy; the following criteria:

(1) Minimum population of 5000.
(2) At least 75% of male working population was non-agricultural
(3) The density of population of at least 400 per sq km (i.e. 1000 per sq Mile).

The Director of Census operations of each state is given some discretion to include in the list of towns, some places which did not fully satisfy the above criteria but had other distinct urban characteristics and to declassify certain undeserving cases from the list of towns in consultation with the State Government. New colonies of industrial attachments belong to this classification mostly with population below 5000 but are very negligible compared to the total population.

The urban places are classified into six classes as below 5000', 5000 to 10,000, 10,000 to 20,000, 20,000 to 50,000, 50,000 to 1,00,000 and above 1,00,000. The number of places belonging to the class 'below 5,000' are mostly a subjective classification by Census Directors as mentioned above and their inclusion and declassification at different censuspoints poses certain difficulties in constructing the transition probabilities. As they are very few in number and their inclusion as urban places is mostly of subjective nature, this category has been omitted **fromour**study.

We are concerned about the demonstration of Markov Model to measure the cy size mobility, as a first step to avoid complications, restriction is resorted **to closed** Markov Model. The number of places are so chosen to fit into the system of closed Markov Model as presented in Section 2. The places included in the analysis are so selected that a place found in 1901 continues to hold the status of urban throughout the period of observation. There are about 111 such places holding a continuous status of urban during 1901 to 1971. This kind of selection also implicitly provides an opportunity to demonstrate the use **of** Markov Model to study the internal structure of a 'cohort of cities'. The cohort constituting the number of urban places which continue to held the status of urban during the entire period of observation.

The figures in Table 1 presents the number of urban places in different states (size classes) along with the total population during 1901 to 1971 and the numbers in brackets indicate the cumulative percentages. The five classes I = 5,000-10,000, II = 10,000-20,000, III = 20,000-50,000, IV = 50,000-1,00,000, and V = above 100,000 constitute the different states of Markov Model under consideration. The evidence in Table 1 amply demonstrates the considerable movement among different size classes by cities necessitating to measure their mobility.

In 1901 there were 26 urban places with population 'above 20,000' constituting 24% of urban places embracing 62% of urban population, in 1931, 42 places constituting 38% held a population of 76% whereas in 1971 the total number of urban places with a population of 20,000 and above constitute 62% with a population of 95%. These figures reveal the mobile nature of cities. **On** the two bottom classes (i.e. class I and II) in 1901 had 85 places with 38% of urban population, m 1931, 69 places had 24% of population and in 1971, 28% of the places held a population of only 5%. These two categories had Considerable depletion whereas the other three classes gained by movements. Though backward movements could be possible, most of the movements are to upper classes corroborating the increasing nature of population in the urban places. Out of 18 movements in 1901, 14 were to upper classes and 4 were to

TABLE—TOTAL NUMBER OF URBAN PLACES IN DIFFERENT SIZE CLASSES WITH POPULATION IN TAMIL NADU—1901-71

	Number of Places	Population		Number of Places	Population
1901			1941		
I	36 (32)	280017(11)	I	20 (18)	170866 (4)
II	49 (76)	691153(38)	II	45 (59)	651465 (21)
III	17 (92)	481813 (57)	III	31 (87)	907953 (44)
IV	6(97)	350650 (71)	IV	10 (96)	639637 (60)
V	3 (100)	751872(100)	V	5 (100)	1540245 (100)
	111	2555505		111	3953081
1911			1951		
I	31 (28)	251131 (9)	I	10(9)	82765 (2)
II	49 (72)	694167 (34)	II	38 (43)	545196 (12)
III	22 (92)	641167(57)	III	45 (84)	1427908 (38)
IV	6(97)	354747 (70)	IV	11 (94)	804294 (53)
V	3(100)	813262 (100)	V	7 (100)	2603552 (100)
	111	2754803		111	5463715
1921			1961		
1	31 (28)	247870 (9)	I	5(5)	38992 (1)
II	48 (71)	675381 (33)	II	39 (40)	561285(10)
III	20 (89)	550537 (52)	III	40 (76)	1271570(29)
IV	9(97)	508557 (70)	IV	18 (92)	1192569(47)
V	3 (100)	850852 (100)	V	9 (100)	3394541 (100)
	111	2833192		111	6458957
1931			1971		
I	22(20)	177456 (5)	I	2 (2)	16069 (0)
II	47 (62)	649045 (24)	II	29 (28)	458145 (5)
III	29 (88)	811557(48)	III	42 (56)	1406457 (22)
IV	9(96)	575447 (65)	IV	22 (86)	1362935 (38)
V	4(100)	1160592(100)	V	16 (100)	5347736 (100)
	111	3373097		111	8591342

lower classes, in 1931 out of 19, 5 were to lower classes and 14 were to upper: classes and in 1971 all the 34 movements were to upper classes. This again reveals that as increase in population of urban places are taking place, backward movements disappear leaving the chances to all urban areas to proceed to next size class.

Table 2 provides the transition probabilities characterising the movements of cities by different size classes during 1901-1971. In the data previewed above, the transition matrix for different periods will be as in (1.1) with i, *j* = 1,2,3, 4 and 5. For example in the transition matrix of 1931/1941, P_{11} = 0.7727 ex plains that a city in the class I in 1931 had a chance of 0.7727 to continue in the same class for another decade and a chance *of* P_{12} — 0.2273 *to* move on to class II. Like wise P_{44} = 0.7778 gives a probability of 0.7778 to a city in class IV to continue the same status for another ten years and a chance of 0.1111 to move back to class III, with a probability of 0.1111 to move forward to class V. The other elements of the transition matrices for various periods can be given similar interpretation.

The states (size classes) through which the cities passes my be of three types and the values of *Pii* in the matrix *P* as in (1.1) defines these three types. If every state can be entered from any other and the system can be left, the system is transient and the values of one or more horizontal lines in the matrix will total less than one. If every state can be entered from any other but the system cannot be left, the system is ergodic. In this case the total value of every horizontal line will be one. Thirdly, if one or more states are ergodic, the system is absorbing and in this case all the values except 1 on one or more horizontal lines will be equal 0 and the values on the diagonal P_{11}, P_{22t} , ,, P_{nn} will be 1.0. The transition matrices for all periods fall into this category of absorbing Markov Chain, each matrix having the class V = above 1,00,000 as absorbing class except the transition for the year 1911/1921 containing two states, class IV = 50,000-1,00,000 and class V = above 1,00,000 as absorbing classes. It explains the system as it is expected to do. As we are observing a growing population, it is but natural that all cities will move to the largest class however long the waitingtime need be. Looking at the diagonal elements of the transition matrix, we could observe *that the* elements P_{55} is 1 for all the periods indicating the sojourn of cities into different classes ends in class V. The class V being absorbing, class IV also depicts some stability without much change in the probability of a city being staying in this class at different points of observation. With regard to P_{33}, the stability considerably declines as we move from 1901 to 1971 resulting in greater mobility when rapid population change takes place. With reference to class I and II, chances of staying in it considerably

Table [illegible] MATRIX OF CSD 1901-71 IN TAMIL NADU

	I	*II*	*III*	*IV*	*V*
1901-11					
I	0.7778	0.2222	0	0	0
II	0.0612	0.8367	0.1021	0	0
III	0	0	0.9412	0.0588	0
IV	0	0	0.1667	0.8333	0
V	0	0	0	0	1.0000
1911-1921					
I	0.8387	0.1613	0	0	0
II	0.0816	0.8572	0.0612	0	0
III	0.0455	0.0455	0.7727	0.1363	0
IV	0	0	0	1.0000	0
V	0	0	0	0	1.0000
1921-1931					
I	0.6452	0.3548	0	0	0
II	0.0417	0.7500	0.2083	0	0
III	0	0	0.9000	0.9000	0
IV	0	0	01111	0.7778	0.1111
V	0	0	0	0	1.0000
1931-41					
I	0.7727	0.2273	0.	0	0.
II	0.0638	0.8298	0.1064	0	0
III	0	0.0345	0.8621	0.1034	0
IV	0	0	0.1111	0.7778	0.1111
V	0	0	0	0	1.0000
1941-51					
I	0.4500	0.5500	0	0	0
II	0.0222	0.60CO	0.3778	0	0
III	0	0	0.9032	0.0968	0
IV	0	0	0	0.8000	0.2000
V	0	0	0	0	1.0000
1951-1961					
I	0.4000	0.6000	0	0	0
II	0.0263	0.8421	0.1316	0	0
III	0	0.0222	0.7778	0.2000	0
IV	0	0	0	0.8182	0.1818
V	0	0	0	0	1.0000
1961-71					
I	0.4000	0.6000	0	0	0
II	0	0.6667	0.3333	0	0
III	0	0	0.7250	0.2750	0
IV	0	0	0	0.6111	0.3889
V	0	0	0	0	1.0000

declines, providing greater mobility for cities of this group.

If there was a complete absence of growth, each city would stay in its Size class for theoretically infinite period of time, the more rapid the population growth the shorter the period, for which a particular city would be found in a given size class. The transition probabilities also reveal the degree of mobility from adjacent classes is higher than from more distant classes. Each movement either forward or backward is of one step and not *a* single movement during the entire period of observation is of more than one state. This is a very interesting point of empirical evidence, at least for cities under our observation, a chance of transition like random walk is taking place.

It is evident that the mobility indexes either to compare two periods or a single period at different points of time as reviewed in Section 3 cannot be Calculated because of absorbing nature of the model. Hence resort is made to the Fundamental Matrix of an absorbing Markov Chain to identify the average numberofyearseachcityspendsindifferentsizeclasses.Astheintervalbetweentwoconsecutive **ween two consecutive censuses being 10 years, each element of the fundamental** matrix is to be multiplied by 10 to obtain the number of years of stay of each city in different size classes. A straight forward interpretation can also be pos sible keeping each element of the fundamental matrix as a generation, each generation counting for 10 years,

Table-3 represents the average number of generations each city belonging to A particular size has to stay before moving to another size class. The fundamental matrices for the period 1901-11, 1961-71 do not exist because the matrices $F = (I - T)$ turns out to be singular. The use of generalised inverse under these circumstances are being looked into. The fundamental matrix elements for the rest of the period is presented in Table 3. As two states IV and V being absorbing during 1911-1921, the fundamental matrix for this period is of order 3 x 3. It is seen that the number of generations each size class has to stay In different states considerably vary within a period and between periods covering the entire period of observation.

The figures in Table 3 amply demonstrate that the cities have fluctuating period of stay before moving to higher classes. The average stay during 1911*21 is larger in all groups. We arc aware that the population growth during this term inclusive of famine and First World War is minimal and is not surprising that each city had to wait for longer generations before moving to next size clast. During 1921-31, the stay for different size classes have become smaller. 19-11 is considered to be a dividing line in population growth, a slowly growing population upto 1921, started growing rapidly because of containment of epidemics ami better distribution of food grains. It is also not surprising that a greater

TABLE 3-THE AVERAGE NUMBER OF GENERATIONS SPENT BY DIFFERENT CITY SIZES IN DIFFERENT SIZE CLASSES

	I	*II*	*III*
1911-21			
I	22.0542	27.2491	7.3367
II	15.8546	27.2491	7.3367
III	7.5884	10.9092	7.3367

1921-'31	**I**	**II**	**III**	**IV**
I	3.3827	4.8008	20	9.0009
II	0.5642	4.8008	20	9.000?
HI	0	0	20	9.0099
IV	0	0	10	9.0009
1931-41				
I	8.7979	15.6702	19,3424	9.0009
II	4.3984	15.6702	19.3424	9.0009
III	1.7604	6.271 7	19.3424	9.0009
IV	0.8802	3.1359	9-6712	9.0009
1941-'51				
I	1.9250	2.6469	10.3306	5
II	0,1068	2.6469	10.3306	5
III	0	0	10.3306	5
IV	0	0	0	5
1951-'61				
I	2.0367	8.4422	5	5.5006
II	0.3701	8.4422	5	5.5006
III	0.0369	0.8435	5	5.5006
IV	0	0	0	5.5006

TABLE 4—VARIANCE OF NUMBER OF GENERATIONS SPENT **BY DIFFERENT CITY SIZES** IN DIFFERENT **SIZE** CLASSES

	I	*II*	*III*	*IV*
1911-21				
I	464.3336	715.2643	46.4904	
II	432.0981	715.2643	46.4904	
III	269.5400	464.612	46.49045	
1921-31				
I	8.06	18.2469	380	72.0152
II	2.9345	18.2469	380	72.0125
III	0	0	380	72.0152
IV	0	0	290	72.0152
1931-41				
I	68.6052	229.8849	354.7829	72.1611
II	53.6491	229.8849	354.7822	72.1611
III	26.1162	150.9417	354.7822	72.1611
IV	25.3414	85.3107	270.9232	72.1611
1941-51				
I	1.7807	4.3592	96.3906	20
II	0.2929	11.3653	96.3906	20
III	0	0	96.3906	20
IV	0	0	96.3905	20
1951-61				
I	2.1114	62.8286	20	24.8154
II	1.0004	62.8286	20	24.8154
III	0.1120	12.6871	20	24.8154
IV	0	0	0	24.8154

TABLE 5—CO-EFFICIENT OF VARIATOIN OF NUMBER OF GENERATIONS SPENT BY DIFFERENT CITY SIZES IN DIFFERENT SIZE CLASSES

	I	II	III	IV
1911-21				
I	97.71	98.15	92.93	
II	131.11	98.15	92.93	
III	21635	197.58	92.93	
1921-31				
I	83.92	88.97	97.47	94.28
II	300.61	88.97	97.47	94.28
HI	0	0	97.47	94.28
IV	0	0	170.29	94.28
1931-41				
I	94.15	96.76	97.38	94.38
II	166.53	96.76	97.38	94,38
III	290.30	195.89	97.38	94.38
IV	571.92	294.54	170.19	94.38
1941-51				
I	69.32	78.88	95.04	89.44
II	506.83	78.88	95.04	89,44
III	0	0	95.04	89.44
IV	0	0	0	89.44
1951-61				
I	71,35	93.89	89.44	90.56
II	270.25	93.89	89-44	90.56
III	407.05	422,28	89.44	90,56
IV	0	0	0	90,36

mean number of generations of stay during 1931-41 is noted. Since greater movements had taken place during 1921-31 each city will be crossing the barrier and will be very near to the lower end of the size classes. This will naturally result in larger number of generations of stay during 1931-41. In fact it can be seen that a sustained slow growth upto 1921 does not allow demotions once it reaches class III or IV after 1921. The zero's in the fundamental matrix represents no backward movements under existing conditions of transitions. Thereafter the average stay has become much smaller providing greater mobility. This is also being revealed by looking into the number of movements during 1911 to 1961, Larger the movements, shorter the number of generations of stay and smaller the movements, longer the stay.

Owing to greater variation in the average number of generations of stay, standard deviation of the average number of generations of stay was calculated and is given in Table 4. Whenever we want to compare the variability of two series which differ widely in their averages, we do not merely depend on standard deviation but calculate co-efficient of variation. According to Professor Karl Pearson who suggested this classical measure, co-efficient of variation is percentage variation in the mean, standard deviation being considered as the total variation in the mean. The series having greater co-efficient of variation is said to be more variable than the other and the series having the lesser co-efficient is said to be more consistent than the other. The C. V. calculated from the data is presented in Table 5, revealing a fund of information. It is noted that C. V. for class III and IV are nearly equal representing a homogeneous variation once a city reaches class III or IV. The erratic values for class I and II indicate the frequent forward and backward movements among these classes;.

Acknowledgement

The author is grateful to Prof. K. N. Venkataraman, Professor of Statistics and Prof. A. Ramesh, Professor of Geography for their guidance and constant encouragement.

References

1. Adelman, **IRMA, G., 1958, 'A** Stochastic Analysis of the size Distribution of Firms'. *Journal of American Statistical Association,* No. 53, 893-904.
2. Bartholomew, D. }., 1973, *Stochastic Models for Social Processes,* 2nd Edn. **Wiley,** New **York.**

3. Census of India, 1971, Series 19—*Tamil nadu* Part II—A Director of Census Operations, Madras, India.
4. Kemeny, G. John and Snell, J. Laurice, 1960, *Finite Markov Chains,* Van Nostrand Com., New York.
5. **Hart, P. E.,** and Prais, S. J., 1956, 'The Analysis of Business Concentration', *Journal ofthe Royal Statistical Society* A, 119, 150-191.
6. Preston, L. E. and Bell E. J., 1961, 'The Statistical Analysis of Industry Structure : an Application to food Industries', *Journal of American Statistical Association* 56, 925-932.